Advances in Multi-dimensional Displacement Measurement using Holographic Interferometry

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Outline of Presentation

- Holographic interferometry
- Holographic moiré
- Novel approaches for multiple phase extraction
- Experimental verification
- Conclusion
Holographic Interferometry

\[ I(n', j') = I_{dc}(n', j') \left\{ 1 + \gamma(n', j') \cos \left[ \frac{\varphi(n', j')}{\lambda} \right] \right\} \]

- Compares two different states of the same object
- Displacements, deformations, vibrations, shape change of rough surfaces such as concrete and steel

Important: Only one displacement component is obtained
Why two displacements are important?

- Application in various fields such as fracture mechanics, biomechanics, model verification for large flawed structures, non-destructive evaluations, etc.
- Measurement of strain fields near stationary and growing cracks
- Crack tip opening displacement during crack growth
- Strain measurements near crack-tip at high temperatures
- Study of deformations in concrete during compressive loadings
- Sometimes out of plane motion can introduce significant errors in in-plane displacement
Holographic Moiré

\[ I(n', j') = I_{dc}(n', j')\left(1 + \gamma_1(n', j')\cos[\phi_1(n', j')] + \gamma_2(n', j')\cos[\phi_2(n', j')]\right) \]

\[ \phi_1(n', j') - \phi_2(n', j') = \frac{2\pi s_z \sin \theta}{\lambda} \]

In-plane displacement

\[ \phi_1(n', j') + \phi_2(n', j') = \frac{2\pi s_z (1 + \cos \theta)}{\lambda} \]

out-of-plane displacement
Novel Approaches in Multiple Phase Shifting Interferometry

- Annihilation filter method (AF)
- State-space approach (SS)
- Multiple Signal Classification method (MUSIC)
- Minimum-Norm algorithm (Min-Norm)
- Estimation of Signal Parameter via Rotational Invariance (ESPRIT)
- Maximum-likelihood estimator (MLE)
High Resolution Approach – I

- Design of Annihilation Filter
  - The method draws parallelism between the frequencies present in the spectrum and the phase steps
  - Design an annihilation filter (a polynomial) which has zeros at the frequencies (phase steps)
- The algorithm extracts multiple phase steps in the presence of additive white Gaussian noise
- The denoising procedure is introduced to enhance the reliability in phase step extraction
Design of Annihilation Filter for Holographic Moiré

- Interference equation for moiré fringes is given by
  \[ I(n', f'; n) = I_{dc} + \sum_{k=1}^{K} a_k \exp[-ik(\varphi_1 + n\alpha)] + \sum_{k=1}^{K} a_k \exp[ik(\varphi_1 + n\alpha)] + \]
  \[ \sum_{k=1}^{K} b_k \exp[-ik(\varphi_2 + n\beta)] + \sum_{k=1}^{K} b_k \exp[ik(\varphi_2 + n\beta)] + \eta; \quad \text{for } n = 0, 1, 2, \ldots, N - 1 \]

- The equation is rewritten as
  \[ I_n(n', f'; n) = I_{dc} + \sum_{k=1}^{K} \ell_k u_k^n + \sum_{k=1}^{K} \ell_k^* (u_k^*)^n + \sum_{k=1}^{K} \varphi_k v_k^n + \sum_{k=1}^{K} \varphi_k^* (v_k^*)^n, \]
  \[ \text{for } n = 0, 1, 2, \ldots, N - 1 \]
  \[ \ell_k = a_k \exp(ik\varphi_1), \quad u_k = \exp(ik\alpha), \quad \varphi_k = b_k \exp(ik\varphi_2), \quad v_k = \exp(ik\beta) \]

- Frequencies present in intensity
  \[ 0, \alpha, 2\alpha, \ldots, \kappa\alpha, -\alpha, -2\alpha, \ldots, -\kappa\alpha, \beta, 2\beta, \ldots, \kappa\beta, -\beta, -2\beta, \ldots, -\kappa\beta \]
Design of Annihilation Filter for Holographic Moiré

- The Z-transform of intensity equation

\[
1(z) = \sum_{n=0}^{N-1} I_n z^{-n}
\]

\[
= \sum_{n=0}^{N-1} \varepsilon_n z^{-n} + \sum_{n=0}^{N-1} \sum_{k=1}^{\kappa} \chi_k^{(n)} z^{-n} + \sum_{n=0}^{N-1} \sum_{k=1}^{\kappa} \chi_k^{(n)} z^{-n}
\]

- The polynomial (annihilation filter)

\[
P(z) = (1 - z^{-1}) \prod_{k=1}^{K} \left(1 - e^{i\alpha_k} z^{-1}\right) \left(1 - e^{-i\alpha_k} z^{-1}\right) \left(1 - e^{i\beta_k} z^{-1}\right) \left(1 - e^{-i\beta_k} z^{-1}\right)
\]

\[
= \sum_{k=0}^{4\kappa+1} p_k z^{-k}
\]

Design of Annihilation Filter for Holographic Moiré

- The discrete convolution of intensity fringes and polynomial is given by

\[ \sum_{k=0}^{N} I_{n-k} P_k = D_n \text{ for } \{n = 0, 1, 2, \ldots, N - 1\} \]
Design of Annihilation Filter for Holographic Moiré

- The discrete convolution of intensity fringes and polynomial is given by

\[ \sum_{k=0}^{n-1} I_{n-k} P_k = D_n \quad \text{for} \quad \{n = 0, 1, 2, \ldots, N - 1\} \]
Design of Annihilation Filter for Holographic Moiré

- Selecting only the central rows corresponding to zero on the right hand side of the matrix gives

\[
\sum_{k=0}^{4\kappa+1} I_{n-k} P_k = 0 \quad \text{for} \quad \{n=4\kappa+1, 4\kappa+2, \ldots, N-1\}
\]

- The phase steps can be computed from the roots of the polynomial \(P(z)\)

\[\alpha = \Re\left(\ln v_i / i\right) \quad \beta = \Re\left(\ln v_i / i\right)\]
Phase Estimation and Error Analysis

\[
\begin{align*}
\Phi_1 & \quad \text{SNR=30dB} \\
\Phi_2 & \quad N = 36 \quad \text{SNR=30dB}
\end{align*}
\]
Typical Example

\[ \varphi_1(x, y) \]

\[ \varphi_2(x, y) \]

\[ \varphi_1(x, y) + \varphi_2(x, y) \]

\[ \varphi_1(x, y) - \varphi_2(x, y) \]
Comments on Annihilation Filter Technique

Salient features

- Identifies multiple phases in the presence of noise
- Handles non-sinusoidal waveform
- Allows converging as well as diverging beams
- Arbitrary phase steps can be imparted
- Phase steps are estimated in real time
- Denoising procedure adds robustness in phase estimation

Concerns!

- Requires a denoising procedure
- Works directly on the signal data
- Needs considerable number of data frames
High Resolution Approach – III

Multiple Signal Classification (MUSIC) method

- The method functions by the design of a covariance matrix

\[
R_f = E \begin{bmatrix}
I(t-1) \\
I'(t-2) \\
\vdots \\
I'(t-m)
\end{bmatrix}
\begin{bmatrix}
I(t-1) & \cdots & I(t-m)
\end{bmatrix} = \begin{bmatrix}
r(0) & r(1) & \cdots & r(m-1)
\end{bmatrix}
\begin{bmatrix}
r^*(0) \\
r^*(1) \\
\vdots \\
r^*(m-1)
\end{bmatrix}
\]

\[
r(p) = E[I(t)I^*(t-p)] = \sum_{n=0}^{4k} A_n^2 e^{j\omega_n p} + \sigma^2 \delta_{p,0}
\]

- Covariance matrix follows the following form

\[
A_m \times A = \begin{bmatrix}
a(\omega_0) & a(\omega_1) & \cdots & a(\omega_k)
\end{bmatrix}
\begin{bmatrix}
a(\omega_0) \\
a(\omega_1) \\
\vdots \\
a(\omega_k)
\end{bmatrix}
\begin{bmatrix}
a(\omega_0) \\
a(\omega_1) \\
\vdots \\
a(\omega_k)
\end{bmatrix} = \begin{bmatrix}
A_0^2 & 0 & \cdots & 0 \\
0 & A_1^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & A_k^2
\end{bmatrix}
\]

A. Patil and P. Rastogi, Optics Express 13, 1240-1248 (2005)
High Resolution Approach – III

Multiple Signal Classification (MUSIC) method

- The signal and noise spaces are separated by performing the singular value decomposition of $R_i$

$$\lambda_1 \geq \lambda_2 \geq \cdots \lambda_{4K+1} \geq \sigma^2$$

$$\lambda_{4K+2} \geq \lambda_{4K+3} \geq \cdots \lambda_m \approx \sigma^2$$

- The orthonormal eigenvectors associated with $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_{4K+1}$ is $S_{m \times n} = [s_1, s_2, \ldots, s_{4K+1}]$ : Signal subspace

- The orthonormal eigenvectors associated with $\lambda_{4K+2} \geq \lambda_{4K+3} \geq \cdots \lambda_m$ is $G_{m \times (m-n)} = [\Phi_1, \Phi_2, \ldots, \Phi_{m-4K+1}]$ : Noise subspace

- MUSIC uses the fact that the noise subspace is orthogonal to the $\{a(\omega_k)\}_{k=0}^n$

$$\sum_{k=m-n}^m a^\ast(\omega) G_k = 0$$

$$a^\ast(\omega) G G^\ast a(\omega) = \|G^\ast a(\omega)\|^2 = 0 \text{ for } m > n$$
High Resolution Approach – III

Design of a sample covariance matrix

- Forward approach
  \[ \hat{R}_f = \frac{1}{N} \sum_{t=m}^{N} \begin{bmatrix} I^* (t-1) \\ I^* (t-2) \\ \vdots \\ I^* (t-m) \end{bmatrix} \]

- Forward backward approach
  \[ \hat{R}_f = \frac{1}{2N} \sum_{t=m}^{N} \begin{bmatrix} I^* (t-1) \\ I^* (t-2) \\ \vdots \\ I^* (t-m) \end{bmatrix} \]
High Resolution Approach – III

$\alpha = 45^\circ$ $\beta = 60^\circ$ $\kappa = 2$

- **Forward approach**

- **Forward backward approach**
Phase step estimation in Holographic Interferometry
Phase Step Estimation in Holographic Moiré

Moiré Fringe

Wrapped Phase

Unwrapped Phase Map

Wrapped Phase

Unwrapped Phase Map
Concluding Remarks

Salient features

- High resolution approach is applied to optical metrology
- Multiple wholefield displacement components can be measured simultaneously using temporal techniques
- Multiple PZTs can be allowed in an optical configuration
- Practical implementation of the concept has been demonstrated