Image Formation of Hologram Reconstruction by Liquid Crystal on Silicon Device

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Introduction
Technical performance of SLM device

Basic structure of SLM device

Holographic film v.s. SLM

<table>
<thead>
<tr>
<th>Technical data</th>
<th>Holographic film</th>
<th>SLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaging process</td>
<td>Wet/Dry-chemical process</td>
<td>Electro-optical conversion</td>
</tr>
<tr>
<td>Display type</td>
<td>Transmission/Reflection</td>
<td>Transmission/Rreflection</td>
</tr>
<tr>
<td>Display area</td>
<td>&gt;50-60 cm²</td>
<td>1.5 x 0.7 cm²</td>
</tr>
<tr>
<td>Resolution</td>
<td>&gt;3,000 lps/mm</td>
<td>6.4 µ (3.74 µ)*</td>
</tr>
<tr>
<td>Structure</td>
<td>Continues</td>
<td>Pixelated</td>
</tr>
<tr>
<td>Operation</td>
<td>Static</td>
<td>Dynamic &amp; Reconfigurable</td>
</tr>
</tbody>
</table>

*Jasper Display: http://www.jasperdisplay.com/tw/
Introduction

- SLM pixel size and viewing angle

Optical reconstruction of digital/computer hologram

Optical reconstruction for 3-D virtual image

Wavelength ($\lambda$): 532 nm
Pixel size ($\Delta x$): 6.4 μm
Viewing angle: $2\theta_{\text{max}} = 2 \sin^{-1} \left( \frac{\lambda}{2\Delta x} \right) \approx 5^\circ$
SLM-based holographic display

- Phase hologram

Coordinate for LCoS-based holographic display

Phase hologram formation
- Diffracted light of "virtual object" pass through SLM-based holographic display system
- Light field formed by a point source $P$ of the "virtual object", in the plane $z=0$

\[
\psi(x, y, \xi, \eta) = \frac{\exp(jkd)}{f} \delta(x - \xi, y - \eta) \exp \left( \frac{jk}{2d} \sqrt{(x - x_0)^2 + (y - y_0)^2} \right) dx_0 dy_0
\]

The expression of a phase hologram with unitary amplitude

\[
\psi'(x, y, \xi, \eta) = \frac{1}{f} \exp \left( \frac{jk}{2d} \sqrt{(x - \xi)^2 + (y - \eta)^2} \right)
\]

Phase hologram on LCoS device

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SLM-based holographic display
- Image formation and properties

- The phase hologram formed in LCoS
  \[ H_s(x, y; \xi, \eta) = w'_s(x, y; \xi, \eta)w(x, y) \]

- The window function of SLM device
  
  Aperture of the SLM
  \[ w(x, y) = \text{rect}(\frac{x}{N_s \Delta x}) \cdot \text{rect}(\frac{y}{N_s \Delta y}) \]

  Pixel of the SLM
  \[ w_s(x) = \text{rect}(\frac{x}{\Delta x}) \cdot \text{comb}(\frac{x - \Delta x/2}{\Delta x}) \]
  \[ w_s(y) = \text{rect}(\frac{y}{\Delta y}) \cdot \text{comb}(\frac{y - \Delta y/2}{\Delta y}) \]

Device structure and coordinate definitions

- Image formation and properties
  - LCoS parameters
  - Pixel number \((N_x, N_y)\), pixel size \((\Delta x, \Delta y)\), and fill factor \((\alpha, \beta)\)
SLM-based holographic display

Impulse response

According to the Fourier optics, the reconstructed image in the Fresnel plane is

\[ h(x, y; \xi, \eta) = \frac{1}{2d} \exp \left[ \frac{jk}{2d} \left( \xi \Delta x + \eta \Delta y \right) \right] \]

\[ = \exp \left[ -\frac{jk}{2d} \left( x^2 + y^2 \right) \right] \ast \left[ \exp \left( -j \frac{2 \pi}{\lambda} \left( \frac{x^2 + y^2}{2} \right) \right) \right] \]

A simplified result in the case of the zero diffracted order

\[ h_0(x, y; \xi, \eta) = \frac{1}{2d} \exp \left[ \frac{jk}{2d} \left( \xi \Delta x + \eta \Delta y \right) \right] \ast \left[ \exp \left( -j \frac{2 \pi}{\lambda} \left( \frac{x^2 + y^2}{2} \right) \right) \right] \]

\[ = \frac{1}{2d} \exp \left[ \frac{jk}{2d} \left( \xi \Delta x + \eta \Delta y \right) \right] \ast \left[ \exp \left( -j \frac{2 \pi}{\lambda} \left( \frac{x^2 + y^2}{2} \right) \right) \right] \]

The reconstructed image

- Modulated by sinc function of SLM array size (Large array size for high quality reconstruction)
- 3-D object can be considered as a collection of many point sources in space, so this analysis method can be applied to 3D reconstruction
SLM-based holographic display

- Depth of focus

The depth of focus as a function of $[\text{sinc}(x/T_s)]^2$

- The reconstruction distance $d' \neq d$
- The light intensity distribution at image plane is proportional to $[\text{sinc}(x/T) \text{sinc}(y/T)]^2$

\[ 2T = 2T_s = \frac{\lambda d}{N_s \Delta x} = \frac{|d - d'|}{d} N_s \Delta x \]

Depth of focus:
\[ d_s = \frac{2\lambda d^2}{(N_s \Delta x)^2} \]

Recording configuration:
- Wavelength ($\lambda$): 532 nm
- Pixel number ($N_s$): 1024
- Pixel size ($\Delta x$): 6.4 $\mu$m
- Diffraction distance ($d$):
  - $d = 400$ mm \( \Rightarrow d_s = 3.96 \) mm
  - $d = 1200$ mm \( \Rightarrow d_s = 35.67 \) mm

- The depth of focus depend on diffraction distance and device parameters
- A long diffraction distance has large depth of focus
- 3D reconstruction image can be resolved in the depth of focus
Experiments

- Setup and Procedure

Experimental setup of the LCoS-based holographic display

- Light source: DPSS Laser
  - Wavelength: 532 nm
- SLM: liquid crystal on silicon device (LCoS)
  - Size: 1920x1080, 6.4 μm
- Focal length of Lens: 300 mm (L₁)
  - 150 mm (L₂)
- CMOS sensor: 2560x1920, 2.2 μm
- BP: band-pass filter

Optical reconstruction

Spatial frequency plane

Image plane
Experimental
Spatial resolution

Target to be recorded

<table>
<thead>
<tr>
<th>Number</th>
<th>Line Pairs / mm</th>
<th>Line width (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>78.1</td>
<td>6.4</td>
</tr>
<tr>
<td>2</td>
<td>39.1</td>
<td>12.8</td>
</tr>
<tr>
<td>3</td>
<td>26.0</td>
<td>19.2</td>
</tr>
<tr>
<td>4</td>
<td>19.5</td>
<td>25.6</td>
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<tr>
<td>5</td>
<td>15.6</td>
<td>32.0</td>
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<tr>
<td>6</td>
<td>13.0</td>
<td>38.4</td>
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<tr>
<td>7</td>
<td>11.7</td>
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<tr>
<td>8</td>
<td>9.8</td>
<td>51.2</td>
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<td>9</td>
<td>8.7</td>
<td>57.6</td>
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<tr>
<td>10</td>
<td>7.8</td>
<td>64.0</td>
</tr>
</tbody>
</table>

Optical reconstruction image

Reconstruction plane

Cross-section profile at “line pair 4”
Experimental results

Depth of focus

Object planes at different depths

Optical configuration:
- Reconstruction distance: $d = 263$ mm
- Depth of focus: $d_f = 1.4$ mm
- Spacing between sectors $d_v/3 = 0.47$ mm

Reconstruction at different distances by shifting camera

Camera positions:
- Reconstruction distance: $d = 263$ mm
- Shift distance $\Delta d = 0.5$ mm
Potential Applications

- SLM-based holographic display with paraxial optical system

- Ray matrix optics and scalar diffraction theory can be applied to SLM-based holographic display architecture combining with paraxial optical systems.
- Paraxial optical system can be a telescopic imaging, 4-f processing system et al.
- The image formation and properties can be also obtained by the proposal method.


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Summary

- This work describes the image formation and properties of the SLM-based holographic 3D display.

- The impulse response and the depth of focus of hologram reconstruction depend on both optical configuration and SLM device structure.

- This approach can be applied to analyze a more general case of SLM-based holographic display with paraxial optical system.
Thank you for your attention

Collaboration work

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